Economical Dynamic. Marketing.

Nabiyev Samat
Kazak British Technical University, Almaty, Kazakhstan
E-mail: ineshnabi@mail.ru

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Abstract

For any business, one of the most important issues is to determine the volume of future sales. This work is devoted to this topic. The approach to solve it is taken by analogy with analytical mechanics: we will consider the commodity market as a dynamic system, the driving forces in which are the marketing strategies of market participants. The main result of this work is the *Basic Equation of Economic Dynamics*. The resulting equation will allow businesses to properly plan and manage their sales, understanding the dynamic dependence of their marketing activities and the results of these actions.

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1 Introduction

A good Marketer knows the marketing tools that increase sales; knows how, where and which one to use. Having reached certain heights, he begins to think about how to reach the next stage of development and do his work more consciously and prudently. Not just experiment with different types of marketing and then analyze their results; but to figure out exactly which marketing activities lead to which results even before you conduct them. If we are talking about calculations, then (Higher) Mathematics is indispensable. To solve this problem, this Marketer will be helped by the *Basic Equation of Economic Dynamics (BEED)*:

$$\frac{\frac{d}{dt}X(t)}{X(t)} = F(\text{Marketing Expenses})$$

where $X(t)$ is the volume of sales, F is a function depending on marketing activities.

The expression $\frac{\frac{d}{dt}X(t)}{X(t)}$ is the sales growth rate. Therefore, *BEED* talks about how the rate of sales growth depends on marketing activities.

2 Literature review

The defining of sales in the world of business and economic analytics is, if not the most discussed and studied, then one of the main ones in economic science. However, today there is not a single universally recognized theory that entrepreneurs would follow. The very presentation of all the results made in this area can be a separate work, which is also quite a lot. The presented article does not contain any ideas of other authors, on the basis of which the results were obtained.

Attempts to create a mathematical model of the Economy and its individual fragments have been made many times, but to this day there is not one of them that can be considered successful with confidence.

Roughly, these attempts can be divided into two main approaches: using the apparatus of *Mathematical Statistics* and using *Game Theory*.

The first approach is based on the fact that economic phenomena are accepted as random processes. Therefore, a description is given of the dynamics of such economic indicators as – the cost of goods, shares, etc.; cash flow generated by the business and much more by the stochastic process formula.

For example.

$dS = B\,dt + C\,dW$, where S is the cost of goods or assets, t is the time, W is the Wiener process (Brownian motion process), B and C are the characteristic functions of the process.

The second approach is based on the *Theory of Games*, which originated in the early twentieth century as a mathematical model of gambling. In short, it consists in the fact that there are N participants (players), each of which has an objective function F_i (payoff) $i = 1, \dots, N$, which they seek to maximize. At the same time, the participants have a choice of certain behavioral strategies $\sigma_j, j = 1, \dots, N$, on which the values of the objective functions $F_i = F_i(\sigma_1, \dots, \sigma_N)$ depend.

The first model is 213nsuccesssful in that it ignores the fact that a person is able to analyze the past, make a planning for the future and, in accordance with this, correct his behavior in the present. The random process model is suitable for such phenomena as the spread of viruses and infections, corrosion of metals, and the like; but where there is human participation, it is hardly applicable. The only place where this model has "taken root" is actuarial mathematics, which takes into account the statistics of deaths, accidents, traffic accidents, etc. for calculating premiums and costs for pension and insurance payments.

The second model assumes a conscious choice of behavior by the participants in the process. Within the framework of Game Theory, it is accepted that the behavior of a participant in economic relations is dictated by the maximization of its objective function, and the usefulness and uselessness of such behavior and the chosen objective function are beyond mathematics.

But game theorists who create such economic theories misunderstand the motivations of market participants and the benchmarks on which they rely. They do not use such tools of financial analysis as profitability, turnover, liquidity and stability, which modern business breathes. Also, such models are usually considered as static systems without taking into account the dynamics of changes and the influence of past actions.

Generally, theories of economics are created as people who come from an entrepreneurial environment, i.e. “practitioners” and people who came from science, i.e. “theorists”.

Practitioners come from their experience, but they lack scientific capacity and methodology. Therefore, their works of Financial Analysis and Project Analysis use an apparatus that, from the point of view of modern mathematics, is no more complicated than arithmetic. In turn, theorists, when constructing their theories, poorly understand the processes of management, i.e. how decisions are made, what key parameters (other than profit) are important when making these decisions, etc. Therefore, the theories of theorists are too abstract and out of touch with reality. A symbiosis of practice and theory is needed.

3 Methodology

For a correct analysis, it is necessary to solve a methodological problem: to determine what is the starting point, what processes need to be considered, what they are; and also, to determine what hardware solutions to solve these problems.

It is necessary to understand the practice of business and how the business lives, i.e. laws of incentive and motivation, financial and project analysis, organization of the production process, logistics, marketing, financial reporting, etc., but at the same time be able to use the gigantic mathematical apparatus that already exists, such disciplines as the theory of differential and variational calculus, the theory of functional analysis, tensor analysis, optimal control theory, etc.

The Basic Equation of Economic Dynamics based on the fact that the system under consideration is a dynamic system, by analogy with analytical mechanics, answers the question Why Sales occur (movement), which is the driving force behind sales.

Usually speaking about the volumes of future sales, all analysts talk about forecasts of these volumes. In other words, the determination of future sales volumes is based on stochastic, probabilistic models. However, probability theory as an analysis tool is used when there is a large uncertainty factor, when it is not possible to calculate all or at least the main factors affecting the result. In the case of Goods Market, these factors can be determined in most cases. In a world where information on many things is available, you can find all the data on all major market operators: their sales volumes, used tools of marketing activities and costs of them and more. For this reason, dynamic analysis and calculation of future sales volumes will give a much better result in comparison with stochastic models.

4 Results and Discussion

Let us show why a formula which describes future sales exists. As an example, let's simulate a sales market in a Shopping Center.

A person who enters the Shopping Center, mentally, somewhere in the subconscious, has already spent a certain amount of money just by entering the shopping center. Otherwise, why did he decide to go there? At the same time, this amount is quite certain, say 5,000 tenge. That is, a person will not leave the mall until he spends them, and most likely he will not spend more than these 5,000. There are cases when a person goes with a specific goal to buy this or that thing, but mostly people go to the mall without a specific goal. More precisely, this goal is simply to spend 5,000 tenge.

So, the mall is visited by, say, 10,000 people a day with an average budget of 5,000 tenge for all sorts of purchases. We get that the total intention to spend the money of all buyers per day in the mall is 50 million tenge. Now the question arises: how will this money be distributed among the sellers located in the mall? And this is where Marketing begins. Marketers enter the arena with their marketing tools to influence the buyer. The better the work of a marketer, the higher the sales, the more a piece of 50 million tenge will be received by the company where this marketer works. In other words, the share of the seller in the total basket of 50 million tenge depends on the marketing activities carried out by the seller. A struggle begins between sellers for the buyer, for his loyalty, for his preferences. And this struggle translates into costs that all companies bear for attracting customers.

Companies make discounts, i.e. receive an expense in the form of lost income. Companies located in a more advantageous location in the mall have a marketing advantage; however, the owners of the shopping center, knowing this, take higher rents from them, i.e. you have to pay for this advantage. In other words, any marketing requires costs.

This fact correlates with the fact that nothing happens by itself, and if you want to get some kind of result (sales of goods), then you need to make an effort for this. All of these efforts come at a cost.

Let's sum up all marketing expenses of all companies located in the mall. Let's assume that this amount was 5 million tenge, i.e. 10% of all sales. At the same time, it is logical to assume that companies invest in marketing in proportion to their market share. That is, if a company occupies 10% of the market (10% * 50 million = 5 million), then it invests 10% of the total amount of all marketing expenses of all companies in marketing (10% * 5 million = 0.5 million). Indeed, companies monitor the behavior of competitors and therefore the arsenal of marketing tools for all companies is the same. Therefore, the efficiency of their investments in these marketing tools should be the same.

If the company decides to increase its market share, then it must invest more in marketing to achieve its goal.

Let's take a certain company in the mall. Let's denote its market share as λ. Then, in order to be at this level of sales, this company must invest in marketing an amount equal to $\lambda \cdot I$, where I is the sum of all investments in marketing of all companies, i.e. $I = 5$ million tenge. Now, if a company decides to increase its market share by $d\lambda$ over a period of time dt, i.e. to the level $\lambda + d\lambda$, then it must invest more than $\lambda \cdot I$ in marketing. Moreover, this "extra" payment must be proportional to the "jump" $d\lambda$. Denote by inv the investment in the marketing of this company. Then

$$inv = \lambda \cdot I + d\lambda \cdot M(dt)$$

where the multiplier $M(dt)$ is the "price" for a 1% jump over time dt. If we now assume that the "price" of the jump per unit of time is M, then $M(dt) = M/dt$. Then the final formula will take the form:

$$inv = \lambda \cdot I + \frac{d\lambda}{dt} \cdot M$$

This is the *Basic Equation of Economic Dynamics*, which gives the mathematical functional dependence of market share and investment in marketing. This equation is true for all mall companies.

Let $X^i(t)$ be the sales volume of company i at time t. Let us denote the total volume of the market as $X^{\text{total}}(t)$. Then the market share of this company, which we denote by $\lambda^i(t)$, will be equal to:

$$\lambda^i(t) = \frac{X^i(t)}{X^{\text{total}}(t)} .$$

Then *BEED* could be written as

$$inv^i(t) = \lambda^i(t) inv(t) + M \frac{d}{dt} \lambda^i(t) .$$

And in terms of volume of sales it will be

$$inv^i(t) - \frac{X^i(t)}{X^{\text{total}}(t)} inv(t) = M \frac{d}{dt} \frac{X^i(t)}{X^{\text{total}}(t)} .$$

Let us now carry out a series of transformations: multiply both sides by $\frac{1}{M}\frac{X^{\text{total}}(t)}{X^i(t)}$, then

$$\frac{X^{\text{total}}(t)}{X^i(t)}\frac{inv^i(t)}{M} - \frac{inv(t)}{M} = \frac{X^{\text{total}}(t)}{X^i(t)}\frac{d}{dt}\frac{X^i(t)}{X^{\text{total}}(t)(t)} = \frac{\frac{d}{dt}X^i(t)}{X^i(t)} - \frac{\frac{d}{dt}X^{\text{total}}(t)}{X^{\text{total}}(t)}.$$

Now, if we rearrange the terms, we get:

$$\frac{\frac{d}{dt}X^i(t)}{X^i(t)} - \frac{X^{\text{total}}(t)}{X^i(t)}\frac{inv^i(t)}{M} = \frac{\frac{d}{dt}X^{\text{total}}(t)}{X^{\text{total}}(t)} - \frac{inv(t)}{M}.$$

In the last equality, the right side does not depend on the volume of sales of the company, i.e. it is the same for all companies.

Now note that the expression $\frac{\frac{d}{dt}X^i(t)}{X^i(t)}$ is the rate of sales growth. The expression $\frac{X^{\text{total}}(t)}{X^i(t)}$ is the reciprocal of the company's market share, and what we previously called "investment" in sales $inv^i(t)$ in the language of accounting is called implementation costs.

Thus, we have obtained the dependence of the sales growth rate on the company's market share and implementation costs.

However, it should be noted that $inv^i(t) \neq$ implementation costs. This is because different marketing tools have different effectiveness.

The result of the sales expenses incurred should be a change in sales volume, but 1 tenge invested, say, in advertising and 1 tenge used as a discount for the client, will give a different effect on sales volume. However, in accounting they are equal.

Let the company allocate a certain budget for the costs of promoting the product, which consists of various types of costs:

$Budget = Expense_1 + \cdots + Expense_n$.

Then, investment in sales is a function of these costs:

$inv^i(t) = f(Expense_1; \dots; Expense_n)$.

In management accounting, expenses are recorded in relation to the required revenue, i.e. sales volume $X^i(t)$:

$\sigma_{\text{E}x\ 1} = \frac{Expense_1}{X^i(t)}, \dots, \sigma_{\text{Ex n}} = \frac{Expense_n}{X^i(t)}$

Therefore, one can write that

$$inv^i(t) = f\left(\sigma_{\text{Ex 1}}X^i(t); \dots; \sigma_{\text{E}x\ \text{n}}X^i(t)\right).$$

or

$$inv^i(t) = f\left(\sigma_{\text{E}x\ 1}; \dots; \sigma_{\text{Ex n}}; X^i(t)\right).$$

In some cases, "scaling" is not definitive, i.e. there is proportionality

$$inv^i(t) = f(\sigma_{\text{E}x\ 1}; \dots; \sigma_{\text{E}x\ \text{n}})X^i(t).$$

$$\frac{inv^i(t)}{X^i(t)} = f(\sigma_{\mathrm{E}x\ 1}; \dots; \sigma_{\mathrm{E}x\ \mathrm{n}}) \ .$$

Here we can recall the Pareto principle (20/80): 20% of the effort brings 80% of the result. Since the vector $\boldsymbol{\sigma}^i(t) = (\sigma_{\mathrm{E}x\ 1}; \dots; \sigma_{\mathrm{E}x\ \mathrm{n}})$ indicates the "efforts" of the enterprise applied to the market to promote its products, these efforts give different results. But as the principle itself says, all efforts are necessary regardless of their contribution, otherwise there is a redistribution of the effectiveness of the contribution that preserves the ratio of 20/80.

Moreover, if we take into account the 4P marketing theory (Product, Price, Promotion, Place), then the list of expenses and, accordingly, the arguments of the function $inv^i(t)$ must include the cost of production, which reflects the Product factor. The Promotion factors are promotion costs and the Place factors are distribution costs. The Price factor can be viewed as an expense Discount. This means that there is a certain average price level and all enterprises, focusing on this level, set their prices in the form of a discount / markup from it.

Returning to the *BEED*, we see that it will take the form:

$$\frac{\frac{d}{dt}X^i(t)}{X^i(t)} - \frac{X^{\mathrm{total}}(t)}{M} f\left(\boldsymbol{\sigma}^i\right) = \frac{\frac{d}{dt}X^{\mathrm{total}}(t)}{X^{\mathrm{total}}(t)} - \frac{inv(t)}{M} \ .$$

It should be noted that, generally speaking, the vector $\boldsymbol{\sigma}^i$ will include not only direct costs, but also the so-called indirect costs of the company, such as costs associated with ensuring the turnover of assets / liabilities (receivables and payables, goods, fixed assets, etc.), other expenses.

It should also be noted that the sales volume of the company and depends not only on its parameters $\boldsymbol{\sigma}^i$, but also on similar parameters of competitors. Because the $inv^i(t) = f\left(\boldsymbol{\sigma}^i\right)X^i(t)$ and $inv(t) = \sum_j inv^j(t) = \sum_j f\left(\boldsymbol{\sigma}^j\right)X^i(t)$ then

$$\frac{\frac{d}{dt}X^i(t)}{X^i(t)} - \frac{X^{\mathrm{total}}(t)}{M} f\left(\boldsymbol{\sigma}^i\right) = \frac{\frac{d}{dt}X^{\mathrm{total}}(t)}{X^{\mathrm{total}}(t)} - \frac{X^{\mathrm{total}}(t)}{M} \sum_j f\left(\boldsymbol{\sigma}^j\right) \frac{X^i(t)}{X^{\mathrm{total}}(t)} \ .$$

It is difficult, if not impossible, to derive these *market functions* $f\left(\boldsymbol{\sigma}^i\right)$ analytically. They can be obtained empirically by tracking the statistics of the behavior of enterprises and the population in a particular market. And we theoretically obtained that the search should be conducted in a differential, dynamic, deterministic form.

Here it is appropriate to recall that, almost all formulas in physics, such as spring force, friction force, Coulomb's law, Maxwell's laws, etc. were obtained empirically. For this reason, the next step in determining BEED should be research based on dates of existing markets.

At the same time, in order to correctly determine the parameters on which market functions will depend, it is necessary to use accounting and financial analysis coefficients, such as sales profitability, turnover of goods and other current assets, growth rates of sales, production and stocks, and much more. This will require a deeper dive into the world of financial and accounting reporting. Moreover, it is necessary to translate financial flows and data of accounting postings into the language of dynamic systems using all the means of modern Mathematics.

5 Conclusion

Now, if we assume that we were able to correctly define *BEED* and solve it with respect to $X^i(t)$ (integrate the equation), i.e. were able to determine the value of $X^i(t)$ depending on the volume of marketing activities; then we can EXACTLY calculate the value of the company's future sales volumes. To many people, this will seem impossible. But …

A plane made of iron and weighing several tons flies in the air and does not fall. This once also seemed unbelievable to people, but it became possible. It became possible after people studied the laws of aerodynamics and learned how to calculate the trajectory of an aircraft. Now this is no longer considered incredible, and the fact that the pilot confidently leads his liner along a pre-calculated trajectory seems obvious and simple.

The same is true with marketing. By learning the laws of marketing and learning how to solve the marketing equations, companies, like pilots, can manage their sales volumes. At the same time, we are not talking about sales forecasts, but about how to accurately calculate these volumes. Continuing the analogy with aviation, pilots fly not along forecast routes, but along the trajectory that they calculated in advance.

In fact, very often the most improbable turns into the possible. And this transformation helps to realize Mathematics.

The Basic Equation of Economic Dynamics applies to all commodity markets. However, this equation is different for each market. To determine this equation for a specific market, a specific business, it is necessary to conduct research using the modern apparatus of Advanced Mathematics.

In conclusion, it must be said that the presented work is not completed, but rather is the beginning of many future studies both in the field of Microeconomics of companies and in the field of Macroeconomics, commodity market analysis, and behavioral analysis, etc.

The author of this article plans to continue working in this direction.